Continuous and Discrete Filippov-Type Stability for One-Sided Lipschitzian Inclusions

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The *one-sided Lipschitz (OSL)* regularity of set-valued maps weakens the Lipschitz continuity and allows discontinuities. It is applied e.g., in the analysis of approximations of reachable sets of nonlinear control problems. In this field, Filippov theorems in continuous and discrete time are important tools for stability analysis of control systems or differential inclusions and for convergence proofs for their discretizations.

We study the sensitivity of continuous and discrete time (difference) inclusions with respect to perturbations in the initial set, in the right-hand side set (outer perturbation) and in the state variable argument of the right-hand side (inner perturbation). While in the classical Filippov theorems for Lipschitz right-hand sides, the distance between the trajectory sets of the perturbed and the original system is Lipschitz (of first order) with respect to the perturbations, in the OSL case this distance typically involves the square root of the inner perturbations (and the square root of the step size for the discretizations). These estimates are improved to the first order for *strengthened one-sided Lipschitz (SOSL)* right-hand sides both for differential and inclusions and their Euler discretizations. Thus the Lipschitz rates in the Filippov theorem are extended from the classical Lipschitz set-valued maps to SOSL ones.

We discuss some applications related to infinite time horizon problems, estimates for reachable sets of the (set-valued) explicit Euler method and discrete relaxation results: reachable sets of nonconvex difference inclusions are compared to the ones with the convexified right-hand sides in the case of OSL maps.

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